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The flywheel energy storage systems as an additional source for urban electric vehicles

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Abstract

This paper demonstrates the possibilities and advantages of hybrid systems with kinetic accumulators in modern electric vehicles, operating in urban conditions. With one of most common transport cycles, a comparison has been made between the characteristics of classic and hybrid drive systems in modern electric vehicles. Through computer simulation, the energy flows through the reversible electric machine and the kinetic energy storage system (flywheel) have been determined, and applicability and advantages of such hybrid drives have been proven. In such propulsion systems, increased mileage and significant relief of the electric battery from large energy flows have been found.

Keywords: Flywheel storage systems; Transport cycles; Urban electric vehicles

Introduction

Serious scientific research of the Flywheel as an accumulator of kinetic energy mainly dates from the middle of the last century and is associated with Prof. DW. Rabenhorst.[1,2] He created a laboratory at John Hopkins University–USA for theoretical and experimental establishment of possibilities for real industrial applications. With the development of science and engineering in the fields of high-strength and light materials, high-speed bearings, including magnetic ones, systems for energy conversion, control, vacuuming, creation of electric motor/generators carrying large loads and etc. Now many new products, based on this technology are already on the market.

At present stationary flywheel energy storage systems (FES) with capacity of 10 to 500 [kw] are produced by a number of industrial companies. They usually contain a flywheel or super flywheel with an electric motor/generator ,build into its core, bearing assemblies , preserved vacuum, a control system and an electrical input/output with standard parameters.[3,4] Such FES are usually found along large railway highways[5,6] USA, Spain, and have proven their right to exist due to the possibility of transferring large capacities. They are also used as uninterruptible power supplies (UPS)[5,7] hydraulic elevators, port container handlers, lifts and etc. For the needs of the physical study of the plasma a system has been implemented by means of which a rotor with mass of several to 5E5 kilograms, is braked in a strong electromagnetic field, gaining the name Tokamak.[5] In transport technology for the first time, the flywheel as the main source of energy is used to a propel a trolley between the stops, with intermediate energy loading there, working in so called elementary speed cycles (Figure 1). This is the first application in to a means of transport that become in Switzerland in the seventies of last century with so-called Jyrobus. The flywheel made in the shape of a large steel disk, which together with electric motor is enclosed in a steel casting to reduce aerodynamic losses and safety.

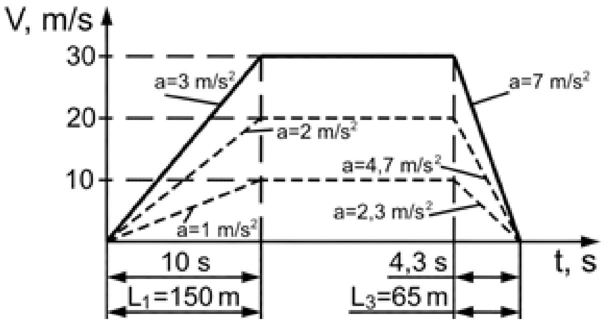


Figure 1 A simple trapezium drive cycle of the vehicle speed profile.

It is more than obvious that FES as the main source of energy in transport technology is practically inapplicable due to the need of intermediate energy charging. Certain prospects exist for means of transport with frequent stops operating in the big City and then only as additional source of energy in combination with electric propulsion or internal combustion engine (ICE). Taxi cars with hybrid drive systems ICE and kinetic energy batter (FES) are already being produced. For the needs of motor sport (Formula one), cars were created on this principle, due to the multiple speed reduction (from 300 t0 150 [km/h]) imposed by the configuration of the racing circuit. The aim of the present study is to substantiate the effectiveness of the application of FES in urban electric vehicles for the most common transport cycles–NEUDC, WLTP3, FUDC, FTP, JCOB and SC03, through computer simulation.

Nomenclature

v - current vehicle speed determined by the drive cycle, $[m/s]$;

m - vehicle mass, $[kg]$;

C_d - vehicle drag coefficient, $[-]$;

δ - coefficient which gives an account on the rotating masses in the vehicle drive line, $[-]$;

f_r - rolling friction coefficient, $[-]$;

α - road gradient, $[-]$;

A_f - vehicle frontal area, $[m^2]$;

k_v - air resistance assigned to an unit of the vehicle mass, $[m^{-1}]$;

η_i - local drive line overall efficiency, $[-]$;

ρ -the ambient air density, 1,202 $[kg/m^3]$;

g is the acceleration of gravity, 9.81 $[m/s^2]$;

P_t - power for active change of the vehicle energy state, $[W]$;

P_f - power for overcoming the road resistance, $[W]$;

P_w - power for overcoming the aerodynamic resistance [W].,

Theoretical considerations

Structural variants of the hybrid electric propulsion system with FES

A flywheel energy source is considered as energy storage. In case of electric vehicles designed especially for urban traffic, the kinetic energy storage is suitable mainly in two structural modifications of the hybrid electrical drive line, as presented on Figure 2 & Figure 3. The first simplified scheme of the hybrid propulsion system for an electric vehicle is shown on Figure 2. The components 1, 2, 3 and 4 represent the conventional electric propulsion system, which allows energy recuperation during vehicle brake modes. The repeated sequences of accelerations and decelerations with high density of the power flows, especially in urban traffic, lead to shortening in the electric battery life. By building in an alternative energy source as FES (pos.6) it is possible to smooth the pick loads over the battery during its charging/discharging modes. The (FES) source has different advantages compared to the electric battery such as increased numbers of charge/discharge cycles or power transfers with higher intensity for examples. The FES coupling to the conventional propulsion system is realized electrically by means of an electric motor pos.5 on Figure 2. Such kind of connection allows high degree of flexibility during driveline design.

The considered hybrid drive line allows the following working modes:

1. The entire recuperation energy during braking is directed to Flywheel, pos.6, at a maximum power output of the electric machine, pos. 5

2. The electric battery is the main energy source for all remaining driving modes if Flywheel's energy level is below a predetermined minimum one.
3. If the energy in FES is over the predetermined minimum value, the FES supports the electric battery with maximum power output of the electric machine 5.

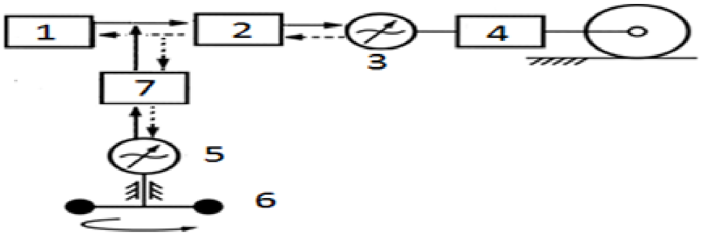


Figure 2 A structural modification of a hybrid electrical drive line contained electric battery 1, inverters 2 and 7, AC electric motor/generators with VFD control 3 and 5, reduction gear 4 and FES 6.

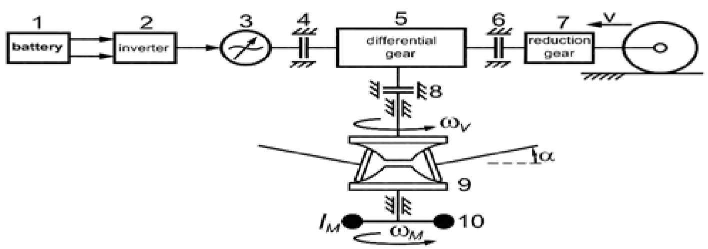


Figure 3 A structural modification of a hybrid electrical driveline consisted of an electric battery 1, an inverter 2, an AC electric motor/generatorr with VFD control 3, clutch-brakes 4, 6, and 8 a differential gear 5, a reduction gear 7, friction variator 9 and FES 10.

This hybrid structure requires many transformations of the energy that means more losses.

The main advantage of the scheme from Figure 3 is that using an appropriate speed control by the control ever of the variator $\alpha = \alpha(\omega_M)$ the output speed ω_V can be kept constant $\omega_V = \text{const}$.[11,15,16] In such a condition it is very easy to control the driveline output speed, i.e. the vehicle speed v will be determined just only by the output speed of the AC electric motor, pos.3.

Estimation of possibilities for energy recovery during braking

The vehicle's energy needed to cover a specific drive cycle with cycle duration of t_c is defined by

$$E_\Sigma = \int_0^t P_t dt + \int_0^t P_f dt + \int_0^t P_w dt \,, \qquad (1)$$

Where different resistances power including the inertial ones could be expressed by the well known relations

$$P_t = m\delta \dot{v} v\,; \qquad P_f = (f_r \pm \sin\alpha) mgv\,;$$

$$P_w = 0.5\rho A_f C_d v^3 \quad (2)$$

It is possible to receive quantity estimation with considerable accuracy of how much energy could be recovered by comparing the necessity energy flows for vehicle acceleration and deceleration over an ideal drive cycle. The last one is considered as a combination of different simple trapezium drive cycles which describe the speed profiles versus time as shown on Figure 1. In case of a discrete speed profile representation the energy required to accelerate a unit of the vehicle mass could be described as

$$E_{\dot{v}\geq 0} = \frac{1}{\eta_1} \sum_{i=1}^{n-1} \left\{ \frac{\delta}{2} \left| v_i^2 - v_{i+1}^2 \right| + g(f \pm \sin\alpha) \int_{t_i}^{t_{i+1}} v\, dt + k_v \int_{t_i}^{t_{i+1}} v^3\, dt \right\} \qquad (3)$$

Similarly the energy required to be absorbed during braking could be presented as

$$E_{\dot{v}<0} = \eta_2 \sum_{i=1}^{n-1} \left\{ \frac{\delta}{2} \left| v_i^{*2} - v_{i+1}^{*2} \right| - g(f \pm \sin\alpha) \int_{t_i^*}^{t_{i+1}^*} v\, dt - k_v \int_{t_i^*}^{t_{i+1}^*} v^3\, dt \right\} \qquad (4)$$

where the time intervals t_i, t_{i+1} $[s]$ correspond to the acceleration modes when the vehicle acceleration $\dot{v}$ is positive or zero; the time intervals t_i^*, t_{i+1}^* $[s]$ correspond to the braking modes with negative acceleration; η_1, η_2 are the drive line efficiencies to and from the driven wheels.

It can be judged from the ratio between the expressions (4) and (3) how much of the exhausted energy for acceleration would be possible to recover in some kind of stored energy. This ratio is considered as a recuperation coefficient k_r, which is presented in the following form:

$$k_r = \frac{E_{\dot{v}<0}}{E_{\dot{v}>0}} = \eta_\Sigma \frac{\sum_{i=1}^{n-1} \left\{ \frac{\delta}{2} \left| v_i^{*2} - v_{i+1}^{*2} \right| - g(f \pm \sin\alpha) \int_{t_i^*}^{t_{i+1}^*} v\, dt - k_v \int_{t_i^*}^{t_{i+1}^*} v^3\, dt \right\}}{\sum_{i=1}^{n-1} \left\{ \frac{\delta}{2} \left| v_i^2 - v_{i+1}^2 \right| + g(f \pm \sin\alpha) \int_{t_i}^{t_{i+1}} v\, dt + k_v \int_{t_i}^{t_{i+1}} v^3\, dt \right\}} \qquad (5)$$

The recuperation coefficient k_r is depicted on Fig.4 as a function of maximum achievable speed for the cycle from Fig.3 at $\alpha = 0$ (zero road gradient angle), $\eta_\Sigma = 0,72$, $k_v = 4.10^{-4}\,[1/m]$, $\delta = 1,1$ and different values of rolling resistance coefficient f_r. The same coefficient k_r is presented on Fig.4, but now the gradient angle is considered as a variable for a constant value of $f = 0.01$. The results represent the optimum scenario when there is no vehicle movement with constant speed, and the simplified drive cycle contains only acceleration and deceleration modes. The presented results analysis shows that for this ideal case there is a possibility to recover 40 to 60 percent of the energy used for acceleration during brake modes instead of transforming it into heat at brake system. The Figure 4 presents the same coefficient as a function of both–maximum speed and gradient angle.

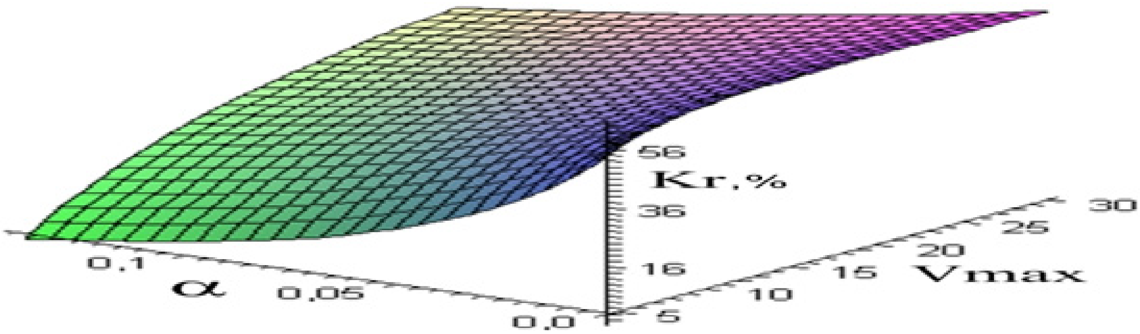


Figure 4 Recuperation coefficient k_r as a function of both the cycle maximum speed according to Fig.3 and the gradient angle α.

In spite of used drive cycle the vehicle movement can be described as a combination of three basic driving modes determined by the necessity direction of energy flow.[13] The condition, which allows those modes to be strictly determined, could be obtained from the differential equation of the vehicle motion in case of free movement on horizontal road and without any disturbances, but only rolling and air drag resistance forces:

$$dv/dt = -\alpha^2 v^2 - \beta^2, \qquad (6)$$

at $\alpha = \sqrt{k_v / \delta}$ and $\beta = \sqrt{f_r g / \delta}$. For $v \geq 0$, i.e. just forward vehicle movement, the equation (6.6) can be integrated in closed form yielding the result

$$v_u(t) = \frac{\beta}{\alpha} \tan\left\{ \arctan\left[\frac{\alpha}{\beta} v(t_0)\right] - (\alpha\beta)t \right\}.$$

Transforming the solution above into the discrete form for every time intervals from the drive cycle leads to the following expression:

$$v_{iu} = \sqrt{f_r g / k_w}\, \tan\left\{ \arctan\left[\sqrt{k_w / f_r g}\, v_{i-1} \right] - \frac{\sqrt{k_w f_r g}}{\delta} \Delta t_i \right\} \qquad (7)$$

The results from (7) can be used to define the operational mode of the vehicle during each discrete time interval from the cycle and the direction of energy flow respectively (Figure 5&6). The different modes are determined by quantity comparison between defined speed at the end of every time interval $i-1, i$ from the drive cycle and estimated speed at the end of the same interval according to expression (7):

$v_i > v_{iu}$ - Necessity for energy supply for fulfillment the defined motion (traction mode);

$v_i < v_{iu}$ - Necessity for energy deprivation for fulfillment the defined motion (braking mode);

$v_i = v_{iu}$ - Without energy transfer (coasting mode).

The described recuperation coefficient (.5) and expressions (.3) and (4) are suitable for a single drive line with just one energy source included because of taking into consideration the overall drive line efficiency. For parallel drive lines with more than one energy source the recuperation coefficient have to be reworked. For that purpose, the recuperation coefficient k_r is transformed into a coefficient of energy recovery at the drive wheels taking into account again the direction of energy flows:

$$k_E = \frac{\left|\Sigma E_{\dot{v}<vui}\right|}{\Sigma E_{\dot{v}>vui}}, \quad (8)$$

Where $\Sigma E_{\dot{v}<vui}$ is the overall energy available for recuperation, but $\Sigma E_{\dot{v}\geq vui}$ is the energy required to fulfill movement over the determined drive cycle. Comparing the expressions (5) and (8) leads to the following relation in case of a conventional electric driveline:

$$k_r = \eta_\Sigma \frac{\left|\Sigma E_{\dot{v}<0}\right|}{\Sigma E_{\dot{v}>0}} \approx \eta_\Sigma \frac{\left|\Sigma E_{\dot{v}<vui}\right|}{\Sigma E_{\dot{v}>vui}} = \eta_\Sigma k_E . \quad (9)$$

The coefficient k_E, expression (8), is a functional characteristic of the drive cycle and of the vehicle resistances coefficients. As k_E is considered as an energy transfer coefficient assigned to the vehicle drive wheels, this coefficient does not depend on the efficiency of the energy transfers among energy sources and the vehicle wheels.

If it is assumed that any predetermined mileage is accomplished by repeating the same drive cycle, the coefficient k_E shows its accumulative effect. Following the same drive cycle decreases the variation of the coefficient k_E and its values tend to a value typical for the end of the first drive cycle as it is shown on Figure 7. This can be explained by energy accumulations in both the numerator and the denominator of the expression (.8). The variation of k_E for the multi-purpose vehicle is presented on Figure 7 for the first (Figure 7a) and the fifteenth (Figure 7b) NEUDC cycle of the sequence. The higher values of k_E at the beginning of the first cycle correspond to the values of the recuperation coefficient k_r (relation 5) because of the short periods of movements with constant speed. During the first 800s of the NEUDC cycle the coefficient k_E tends to a value determined by the small drive cycle ECE-15 (Figure 7a). The alteration of the motion behavior during the last 400s of the cycle changes the variations in the coefficient of k_E. This tendency in k_E behavior to come to a predetermined value as a result of cyclic recurrence of the motion is illustrated on Figure 7b. The presented results allow the analysis for the achievable mileage and necessary initial energy levels of the vehicle energy sources to be carried out just over one cycle independently of the number of the reiterated cycles for the given mileage implementation. Typical values of k_E for the European NEUDC drive cycle are presented in Table 1, which describe six different types of vehicles suitable for usage of electric propulsion system:[13] 1–sport utility vehicle; 2–full size sedan; 3–middle size sedan; 4–compact vehicle; 5–small multi-purpose vehicle; 6–a small bus. Data for k_E are related to the end of the first NEUDC cycle, as it is the value k_E tends to, if sequenced cycles are considered. Final values of the coefficient k_E (by dv/dt ≥0 in denominator (5) for different standard drive cycles are presented in Table 1. The Table 2 represents the final values of the recovery coefficients k_E (by dv/dt ≥0 in denominator) for different standard drive cycles.

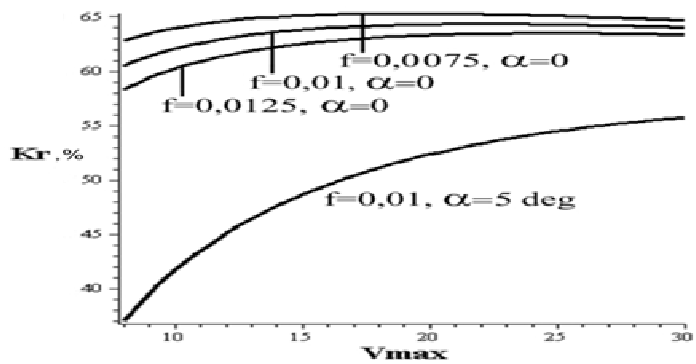


Figure 5 Recuperation coefficient k_r as a function of the cycle maximum speed according to Fig.3 at $\alpha = 0$

Energy recovery during brake modes and its impact on the achievable mileage of a battery electric propulsion system (BEV)

In case of pure electric vehicle propulsion the main challenge is the covered mileage, as the battery charging is still a technical issue. Those systems entire rely on energy recuperation during braking to fulfill the expectations for acceptable mileage covered by a charged battery. In such cases any solutions for increased mileage are of great interest. It is considered a simple structural scheme of conventional electric propulsion system as shown on Figure 8.[15,16] The system contains an electric battery, an inverter, an induction electric motor/generator with variable frequency drive control (VFD), and mechanical driveline of a clutch and a reduction gear. If the recuperation during braking modes is not taken into account, the mileage of such a vehicle can be estimated by energy balance in the following form

$$L = L_C \frac{Q_{bat}\eta_1}{\Sigma E^C}, \qquad (10)$$

where Q_{bat} $[kWh]$ is available energy capacity of the battery $\Sigma E^C = \Sigma E_{\dot{v}>vui}$ $[kWh]$ is the necessary energy to cover one cycle, but L_C is the mileage of one cycle.

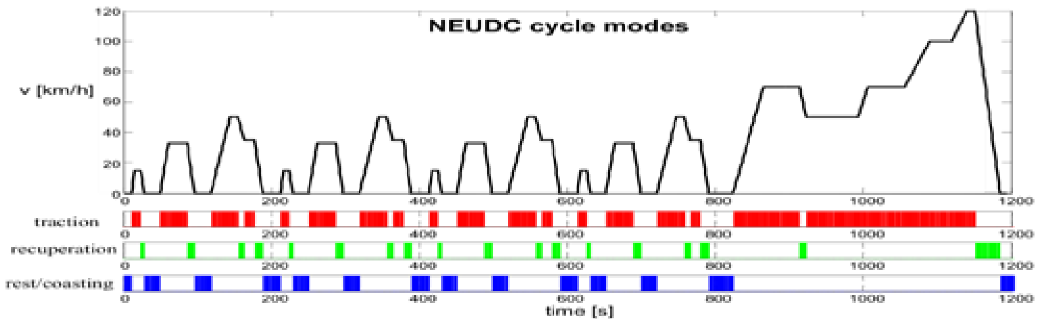
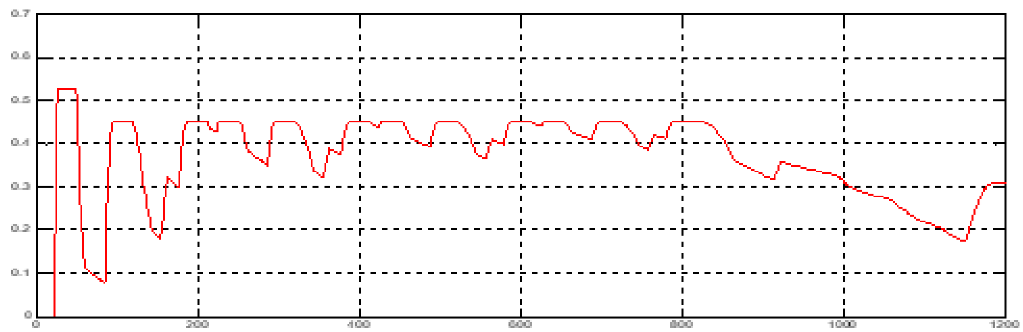
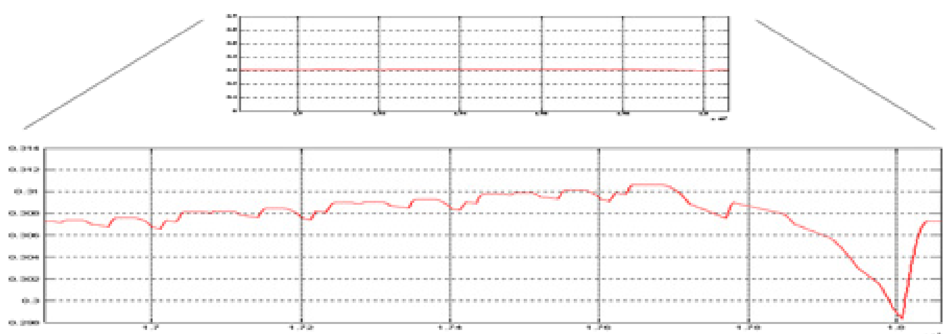


Figure 6 Speed profile of the NEUDC drive cycle and corresponded operation modes.



a) n=1



b) n=15 at different scales of k_E

Figure 7 Evolution of k_E during motion over reiterated NEUDC cycles.

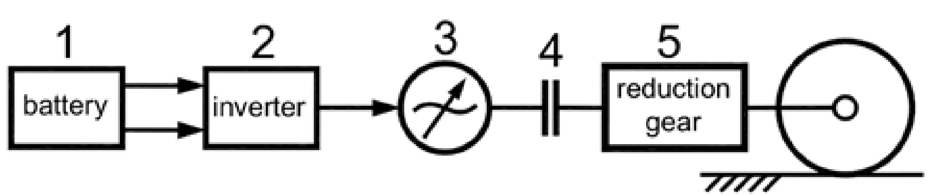
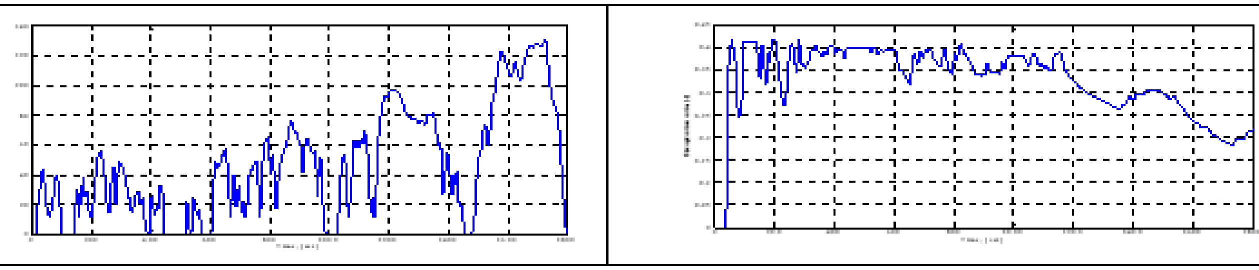


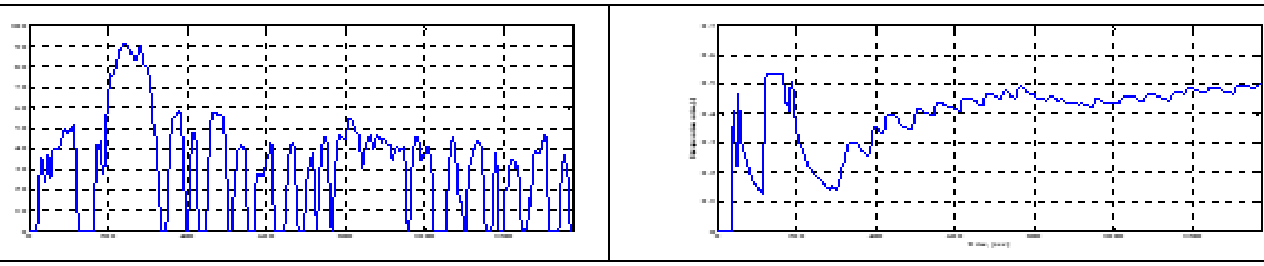
Figure 8 Structural scheme of an electric vehicle contains an electric battery 1, an inverter 2, an AC electric moto/rgenerator with VFD control 3, a clutch 4, reduction gear 5 and drive wheels.

Table 1 Values for k_E in case of different types of vehicles.

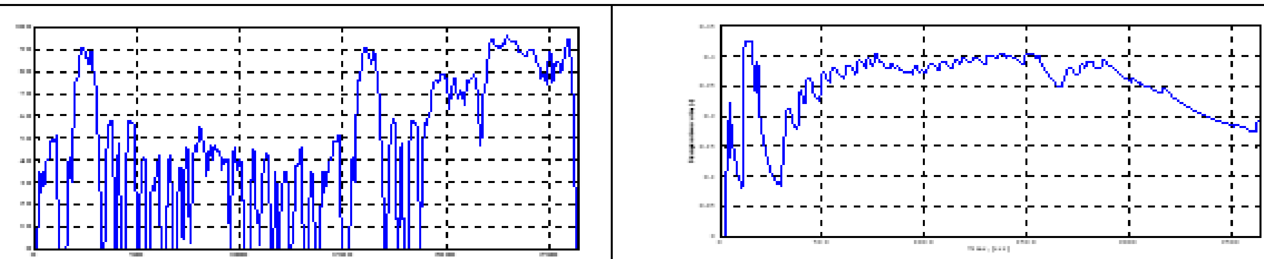
Vehicle type	**1**	**2**	**3**	**4**	**5**	**6**
$A_f C_d [m^2]$	1.2	0.7	0.6	0.4	0.7	1.88
$f_r [-]$	0.017	0.013	0.012	0.008	0.013	0.015
$m[kg]$	2000	1500	1000	750	1700	4436
$k_v [m^{-1}] \times 10^{-4}$	3.6	2.8	3.6	3.2	2.47	2.54
$k_E [-]$	0.2131	0.2805	0.2608	0.3277	0.3073	0.2693



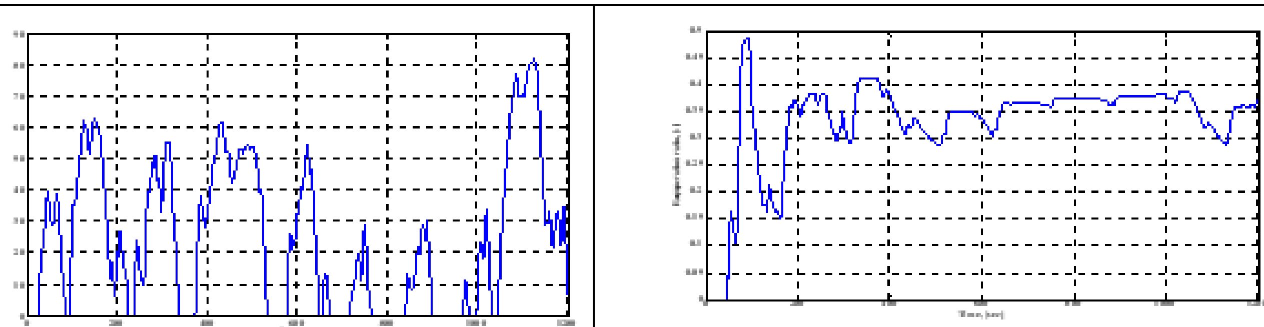
Cycle WLTP3



Cycle FUDC



Cycle FTP



Cycle JC08

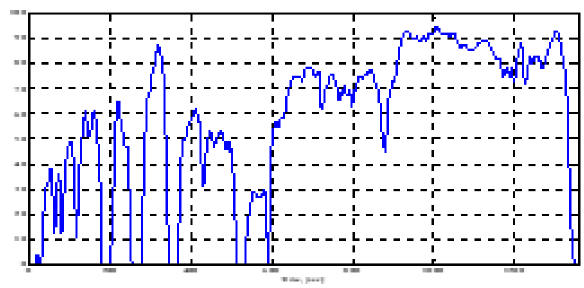
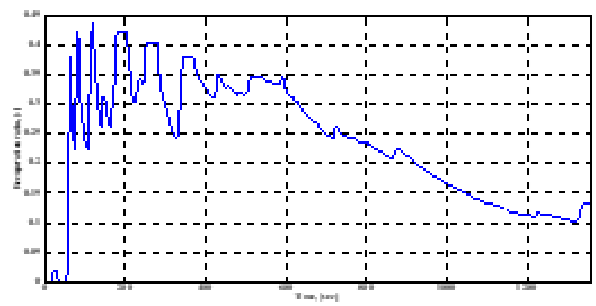
	
Cycle SC03	

Table 2 Coefficient of energy recovery k_E as a function of drive cycle speed profile.

Taking into account the possibilities for energy recovery during brake modes and relation (8) for energy recovery coefficient k_E the energy ΣE^C needed to fulfill one cycle can be expressed by:

$$\Sigma E^C = \Sigma E_{\dot{v} > vui} - \Sigma E_{\dot{v} < vui} = (1-\eta_\Sigma k_E)\Sigma E_{\dot{v} > vui},$$

and after replacing the expression above into the relation (10) the possible mileage is achieved as

$$L^* = L_C \frac{Q_{bat}\eta_1}{\Sigma E^C} = L_C \frac{Q_{bat}\eta_1}{(1-\eta_\Sigma k_E)\Sigma E_{\dot{v} > vui}} = \frac{1}{(1-\eta_\Sigma k_E)} L, \quad (11)$$

where the term $(1-\eta_\Sigma k_E)^{-1}$ represents the possibilities for mileage increase of about 12-17% for overall efficiency of the drive line of $\eta_\Sigma = 0.466$ and the different types of vehicles presented in Table 1 at the same initial energy level of the electric battery (Fig 9. Data from modeling of the electric vehicle i-Mi EV behavior over the NEUDC cycle with energy recovery coefficient of $k_E = 0.26$ are also added. The available data for the energy efficiency concerning the electric vehicle i-Mi EV are obtained by using Japan's drive cycle JP 10-15 [14]. This cycle is an analog of the European NEUDC cycle.A direct comparison between the calculated data according to relation (10) and experimental ones is made after solving the identification problem about rolling resistance coefficient and estimation of the overall efficiency of the i-Mi EV drive line over JP 10-15 cycle. The better result for i-Mi EV is due to the better efficiency of almost $\eta_\Sigma = 0.6$ for its drive line (Figure 9).

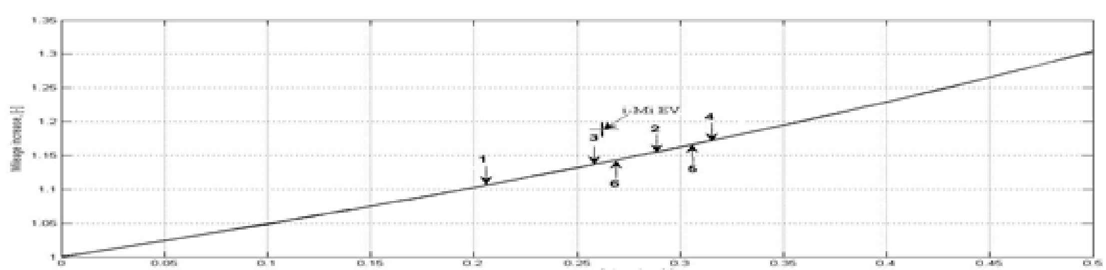


Figure 9 Mileage increase over reiterated NEUDC cycles for the different vehicles presented in Table 6.1 and an overall efficiency of the propulsion system of $\eta_\Sigma = 0.466$.

Initial energy levels for the battery BEV.

In case of conventional electrical propulsion system the coefficient of energy recovery k_E -relations (.8) and (11), can be expressed by the energy stored in the electric battery:

$$k_E = \frac{\Delta E^{ch}_{Bat} / \eta_2}{\eta_1 \Delta E^{disch}_{Bat}} = \frac{\Delta SoC^{ch}_{Bat} / \eta_2}{\eta_1 \Delta SoC^{disch}_{Bat}}, \quad (12)$$

where ΔE_{Bat}^{disch} and ΔE_{Bat}^{ch} are the energy changes of the battery state, which directly correspond to the cycle energy demands $\Sigma E_{\dot{v}>vui}$ and $\Sigma E_{\dot{v}<vui}$, but η_1 and η_2 represent the overall efficiencies of the propulsion system in the both main battery modes–discharging and charging respectively. The battery parameter state of charge $SoC = E_{Bat} / E_{Bat0}$ can be expressed by the first relation of (11), where SoC_{Bat0} describes the initial battery energy (usually $SoC_{Bat0} = 1$).

From the relations (12) and (.3) for known values of k_E, i.e. chosen drive cycle, and overall efficiencies η_1 and η_2 it follows that the current battery state of charge can be expressed as:

$$SoC_{Bat} = SoC_{Bat0} - \left(\frac{1}{\eta_1} - \eta_2 k_E \right) \frac{\Sigma E_{\dot{v}>vui}}{E_{Bat}^{\max}}.$$

If a preliminary condition is attached for full battery discharge at the end of the proposed vehicle mileage

($SoC_{Bat} = 0$), the minimum battery energy state necessary for implementation of the same mileage is calculated as

$$E_{Bat}^{\max} = \left(\frac{1}{\eta_1} - \eta_2 k_E \right) \frac{\Sigma E_{\dot{v}>vui}}{SoC_{Bat0}} \frac{L_d}{L_C}. \qquad (13)$$

The covered distance over one NEUDC cycle is $L_C = 0.8\ km$. The condition of desired mileage of $L_d = 160 km$ only using NEUDC cycle, at $\eta_2 = 0.6624$ and $k_E = 0.3073$ leads to the following results:

1. Conventional electric propulsion system without recuperation - using relation (9), which can be solved against the battery capacity Q_{bat}, or using relation (13) at k_E=0 both variants lead to $Q_{bat} \equiv E^{max}_{bat}$=31.95 kWh.

Conventional electric propulsion system with recuperation - using relation (9), which can be solved against the battery capacity of Q_{bat}, or using relation (.13): both variants lead to $Q_{bat} \equiv E_{Bat}^{\max} = 2\ .3$ kWh.

The latter result concerning the case with recuperation can be transformed into additional vehicle mileage. This results in 15.9% increase in the achievable mileage over NEUDC cycle.

Energy recovery coefficient kE for the hybrid electric propulsion system with FE

The coefficient k_E, relation (.8), which is a drive cycle characteristic, can be expressed by the battery and FES energy levels alteration in the following form:

$$k_E = \frac{\Delta E_{FES}^{ch} / \eta_4 + \left(\Delta \mathring{A}_{Bat}^{ch} / \eta_2 \right) \hat{e}_1}{\eta_1 \Delta \mathring{A}_{Bat}^{disch} + \left(\eta_3 \Delta \mathring{A}_{FES}^{disch} \right) \hat{e}_2},$$

where- η_1 and η_3 are the overall efficiency coefficients of the drive lines from the battery and FES respectively

;- η_2 and η_4 are the overall efficiency coefficients of the drive lines from the driven wheels to the battery and KES respectively;

-- ; $\Delta E^{ch/disch}$ are the energy alterations of the two alternative energy storage devices–the battery and FES-

-coefficients k_1 and k_2 represent the limiting conditions concerning the KES modes of operation.

Using the relation $SoC = E_{Bat} / E_{Bat0}$, which represents the state of charge of the electric battery and a similar expression for KES, the cycle coefficient k_E can be presented in the following form:

$$k_E = \frac{\Delta SoC_{FES}^{ch} / \eta_4 + \left(\Delta SoC_{Bat}^{ch} / \eta_2 \right) k_{Bat/FES} \hat{e}_1}{\eta_1 \Delta SoC_{Bat}^{disch} k_{Bat/FES} + \left(\eta_3 \Delta SoC_{FES}^{disch} \right) \hat{e}_2}, \qquad (14)$$

where the different coefficients k_1, k_2 and $k_{Bat/FES}$ are determined as:

$$k_1 = \begin{cases} 0, & SoC_{FES} \neq 1 \\ 1, & SoC_{FES} = 1 \end{cases}; \quad \hat{e}_{Bat/FES} = \frac{\mathring{A}_{Bat}^{\max}}{\mathring{A}_{FES}^{\max}}; \quad k_2 = \begin{cases} 0, & SoC_{FES} \leq 0.1 \\ 1, & 0.1 \leq SoC_{FES} \leq 1 \end{cases},$$

at

$$SoC_{Bat} = SoC_{Bat0} - \Delta SoC_{Bat}^{disch} + \Delta SoC_{Bat}^{ch} \text{ and}$$

$$SoC_{FES} = SoC_{FES0} - \Delta SoC_{FES}^{disch} + \Delta SoC_{\hat{E}\grave{A}\mathring{A}}^{ch}. \quad (15)$$

The coefficient k_1 is determined by the range where KES is able to store the energy during braking modes. In cases when the FSS is unable to store any additional energy (FSS is fully charged, for example), the energy available for recuperation is transferred to the electric battery and the coefficient switches to a value of $k_1 = 1$. The coefficient k_2 corresponds to mode B (as described above), when it is suitable to use the energy stored into FES to support the electric battery to cover the vehicle energy demands.

Initial energy levels for FES as an alternative energy source

Maximum energy level determination of FES is a complex task. The higher available energy levels allow the higher density of the power density over a time, but also mean mass or velocity increase. The main criterion here would be the condition for cyclic recurrence of the FES usage i.e. what is the optimum for KES energy level for given vehicle and drive cycle that ensures the FES states of charge at the beginning and at the end of the one drive cycle to be almost equal. The condition above can be fulfilled independently by active reduction of the energy flow from FES, which will be discussed further in the next chapter. For this purpose, a power reduction coefficient is introduced. Meanwhile in case of a lack of enough initial data for the hybrid driveline, the most appropriate criterion for KES energy level calculation is the all available energy for recuperation from one cycle to be stored into FES.

FES initial energy level according to a full energy recovery condition at brake modes

The idea that all dissipated energy during braking over one cycle has to be stored into KES is reasonable, as the FES maximum level is very easy to calculate without taking into account KES's discharge processes. In such a condition, the coefficient k_1 in the relation (14) has to be zeroed. By comparing the numerators in relations (.5) and (14), where the relation (14) is rewritten for the FES, it is obtained the following relation

$$\left|\Sigma E_{\text{ṽ}<vui}\right| = \frac{1}{\eta_4}\Delta E_{FES}^{\text{ƅ}} = \frac{1}{\eta_4}\Delta SoC_{FES}^{\text{ƅ}} \mathring{A}_{FES}^{\max},$$

which solution, according to the maximum energy level for KES, is

$$\mathring{A}_{\text{ÊÀÅ}}^{\max} = \frac{\eta_4\left|\Sigma E_{\text{ṽ}<vui}\right|}{\Delta SoC_{FES}^{\text{ƅ}\ \max}}. \quad (16)$$

It is accepted in the theory, developed for KES, that the permissible maximum change of the KES state of charge is $\Delta SoC_{FES}^{\text{ƅ}\ \max} = 0.9$ ($\frac{w_{FES}^{\max}}{w_{FES}^{\min}} \approx 3$ at fixed mass parameters), and using the same initial data for the vehicle and the drive cycle i.e. $\left|\Sigma E_{\text{ṽ}<vui}\right|$=0.468 kWh and η_4=0.7452 the required energy level of KES is $\mathring{A}_{\text{ÊÀÅ}}^{\max}$=0.3875 kWh.

Coefficient of power reduction

Optimum usage of FSS as an alternative energy source requires acceptable FES state of charge over the entire drive cycle. This requirement allows at any energy demand over the drive cycle and chosen strategy of FES usage, or the FES energy levels to be sufficient to cover such a demand.[17] The energy recovery coefficient k_E in case of hybrid electric propulsion system, relation (11), is accepted as a starting point for initial ESS state of charge estimation. The FES state of charge has two upper and lower limits. FES is unable to store the available energy during brake mode at its upper limit which corresponds to fully charged FES at SOC_{FES} =1. On the contrary, if SoC_{FES}=0.1 which corresponds to the minimum acceptable discharged level of FES, it is impossible to use FES as an alternative source during vehicle acceleration modes. In both cases the FES usage as energy buffer storage is compromised which leads to the first constraint condition in relation (14)–k_2=0. While keeping the electric battery as safe as possible, it is also desirable that all available energy for recuperation during brake modes should to be stored into FES i.e. the second coefficient in relation (.14) has to be zeroed (k_1=0). Thus, the relation (.14) for the energy recovery coefficient k_E is transformed as

$$k_E = \frac{\Delta SoC_{FES}^{\text{ƅ}} / \eta_4}{\eta_1 \Delta SoC_{Bat}^{disch} k_{Bat/FES} + \left(\eta_3 \Delta SoC_{FES}^{disch}\right)}.$$

The solution of the above relation according to the electric battery state of discharge ΔSoC_{Bat}^{disch} is

$$\Delta SoC_{Bat}^{disch} = \frac{1}{\eta_1 k_{Bat/FES}}\left(\frac{\Delta SoC_{FES}^{\text{ƅ}}}{\eta_3 k_E} - \eta_3 \Delta SoC_{FES}^{disch}\right),$$

If it is considered that the denominator is a constant i.e. $\eta_1 k_{Bat/FES} = const$, the battery state of discharge has an extremism at

$$\frac{\Delta SoC_{FES}^{ch}{}'}{\eta_4 k_E} - \eta_3 \Delta SoC_{FES}^{disch}{}' = 0$$
,

which leads to

$$\Delta SoC_{FES}^{disch}{}' = \frac{\Delta SoC_{\text{ÊÀÅ}}^{ch}{}'}{\eta_3 \eta_4 k_E} = k_P \Delta SoC_{FES}^{ch}{}' . \qquad (17)$$

The variables $\Delta SoC_{FES}^{ch}{}'$ and $\Delta SoC_{FES}^{disch}{}'$ are equivalent to the intensity of the energy change of the FSS state - ΔE_{FES}^{ch} and ΔE_{FES}^{disch} respectively with accuracy to a constant depending on time. By presenting the variables ΔE_{FES}^{ch} and ΔE_{FES}^{disch} in discrete form which correspond to the drive cycle representation, it is possible to make an analogy to the power transfer to and from the KES i.e. $\Delta E_{FESi}^{ch} \rightarrow P_{FESi}^{ch}$ and $\Delta E_{FESi}^{disch} \rightarrow P_{FESi}^{disch}$, and after substitution into the relation (6.17) it is received

$$P^{disch}_{FESi} = k_{Pi} P^{ch}_{FESi} \qquad (18) ,$$

where $k_P = (\eta_3 \eta_4 k_E)^{-1}$ are transitory values for power reduction coefficient for the electric machine (pos.5 on Fig.1) coupled to KES. After prolonged movement over the same drive cycle the values of k_E tend to the specific value for k_E, which is representative for the given drive cycle (see Table 1 & Figure 7) whence the power reduction coefficient tends to

$$k_P = (\eta_3 \eta_4 k_E)^{-1}, \qquad (19)$$

which is again a typical value for given vehicle and drive cycle.

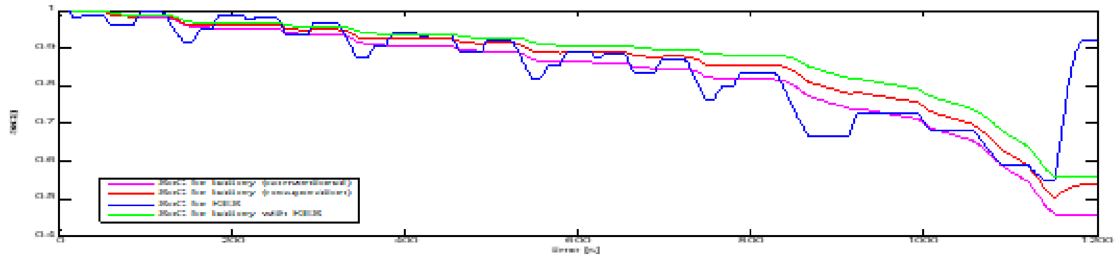
The relation (19) shows that following the idea for adequate energy levels for FES usage during the defined drive cycle, the power flow from FES would be consider as a functional dependence on both the speed recovery of the energy (power transfer for charging), stored in FES, and vehicle driving modes taking into account the energy losses (Figure 10).

Power reduction coefficient impact on energy states of the both energy sources and achievable mileage at fixed initial energy levels of the sources

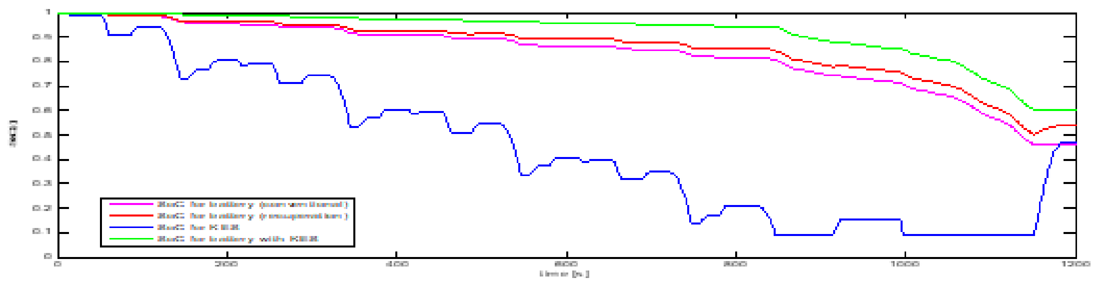
The described power reduction coefficient concerns the maximum power of the electric machine pos.5 coupled to FES (Figure 2). The entire capacity of this electric machine is used at $k_P = 1$ when it is in a generator mode of operation. The increase of the coefficient k_P (which corresponds to k_E decrease) means a decrease of the maximum power, which can be used by FES taking into account the possibilities for energy recuperation. By this way the FES energy state can be controlled over the drive cycle on the expense of the energy level of the electric battery. This control assumes better utilization of FSS energy, as this does not allow prolonged periods of usage of FES at its boundary states (maximum charged, maximum discharged state respectively), which leads to a necessity to switch off the FES branch from the propulsion system. The results from numerical modeling of the energy transfers into the hybrid propulsion system of a multi-purpose vehicle over NEUDC are shown on Fig.10. The energy levels of the alternative energy sources (electric battery and FSS) are presented by their states of charges in the following basic modes of operation:

1. all the energy demands are covered entirely by the electric battery–the conventional electric propulsion system without recuperation;
2. all the energy demands are covered by the electric battery–the conventional electric propulsion system with recuperation
3. the energy demands are covered by the both energy sources –electric battery and FES.

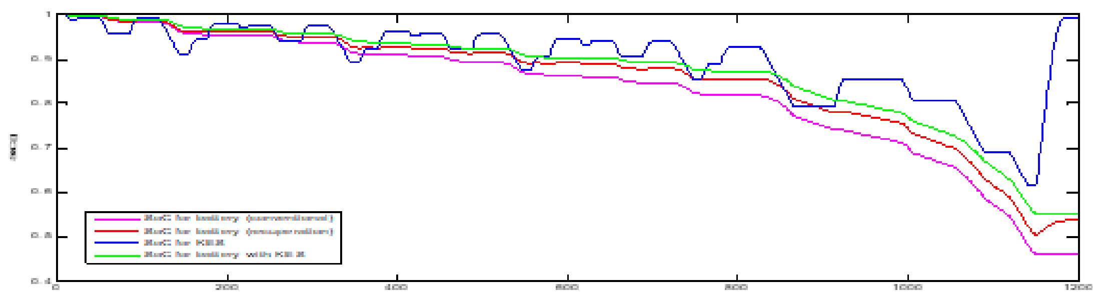
Observing the described possible modes of the hybrid system. It is accepted an initial energy level of 4kWh for the electric battery, which allows mileage over two NEUDC. It is also accepted an initial FES energy capacity of 0.3848 kWh, determined by the condition for full energy recuperation–relation (15). Those initial values give a clearness of the results over just one drive cycle. The energy consumption in the three variants shown on Fig.10 is better than the one in case of conventional electric propulsion with recuperation due to the better efficiency of the power branch to and from FES. The fixed power reduction from FES seems to be the optimal variant for FES usage in case of cyclic recurrence in the vehicle motion.



a) pure electric transmision without recuperation



b)pure electrical transmisionwith recuperation



c) hybrid system with full energy recovery of KES at the end of the cycle

Figure 10 Energy states of the alternative sources over NEUDC drive cy.

FSS influence on achievable mileage

The influence of the power reduction coefficient is illustrated by maximum achievable mileage of the mentioned vehicle until the electric battery becomes fully exhausted. The vehicle motion is modeled over repeated NEUDC cycle at fixed initial energy states for the electric battery and FES fully charged battery and FSS with capacity of 31.95 kWh, and 0.3848 kWh respectively. The results from the modeling are presented in Table 3 and they describe the achievable mileage as a function of the power reduction coefficient k_p, which is practically reciprocal to k_E -- relation (19).

Two typical cases of the initial available energy states for FES–fully charged, and fully discharged respectively it is considered. In both cases the maximum of the achievable mileage is at $k_P \approx 5$, which corresponds to the values from relation (.19).

Numerical modeling is made using the following data: a typical multi-purpose vehicle according to Table 1 - vehicle type 2, (and maximum power of 25 kW for the electric machine coupled to the FES. The main difference here is the fact that only one fifth of this power is used during acceleration and the additional power for covering the energy demands is supplied by the electric battery. The comparative analysis with the conventional propulsion system shows an increase of 22.7% in achievable mileage in case of initially fully charged KES. At low values for k_p, i.e. the maximum usage of the electric machine 5, see Figure 2, the average FES state of charge, which is kept by recuperation, is not enough to cover all the drive cycle energy demands, thus the electric battery has to be used to compensate the shortage of power but with its lower efficiency. On the contrary, at high values for k_p the power reduction of the flow from FES has a negative effect by keeping the FES state of charge at its upper limits, which leeds to the loss of possibility to store recuperation energy into FES. The extremum of the mileage is reached at value which corresponds to the drive cycle energy recovery coefficient, according relation (.19). By comparing the mileage in the both considered cases: fully charged and fully discharged FES, it becomes clear that the available energy inFES, when the FES is fully charged, is enough for covering a mileage of 3.5 to 4 km. This confirms the idea thaFES must be used as a buffer accumulator instead of a main one.

The data presented in Table 4 summarize theFES influence over the entire efficiency of the described multi-purpose vehicle with electric propulsion system over NEUDC cycle. The results are expressed by achievable mileage in case of fixed optimum value for power reduction coefficient k_P of 5 (Table 3).

Table 3 Influence of the power reduction coefficient on the achievable mileage over NEUDC drive cycle.

NEUDC				
Full FES		**Empty KES**		**SOC of FES**
KP	**S [km]**	**KP**	**S [km]**	**to bottom limit**
1	191.05	1	186.75	
2	192.63	2	188.54	
3	193.56	3	189.81	
4	193.71	4	190.29	
5	193.83	5	190.32	
6	190.3	6	186.7	
7	187.95	7	183.67	to upper limit
S_bat=157.75 km; S_bat+rec=181.6 km				

Table 4 General results of motion over NEUDC cycle.

NEUDC	Mileage, km	Duration, h	Number of cycles
Electric battery supply without recuperation	157.75	4.95	15
Electric battery supply with recuperation	181.6	5.65	17
Hybrid supply with fully charged/discharged KES	193.8/190.3	6.0/5.95	18

Conclusion

Based on the theoretical investigations and computing simulations, a real possibility is determined for 25–40% energy recovery during brake modes in city urban drive cycles. An application of a hybrid drive line for electric propulsion system with kinetic energy storage as an alternative energy source is investigated. The results show an increase between 15% and 23% of the achievable mileage of a vehicle with mass of 1700 kg (multi-purpose vehicle-variant 5, Table 1) over NEUDC cyclic recurrence until the main energy source–the electric battery becomes fully discharged.

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Conflicts of interest

Author declares that there is no conflict of interest.

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